UTU Escuela Técnica Buceo	Examen de Matemática 2°	11/07/2008

RESOLUCIÓN

1) i) $P(x) = ax^3 + bx^2 + cx + d$

$P(4) = a(4)^3 + b(4)^2 + c(4) + d = 0 \Rightarrow$
$P(3) = a(3)^3 + b(3)^2 + c(3) + d = 5 \Rightarrow$
$P(1) = a(1)^3 + b(1)^2 + c(1) + d = -9 \Rightarrow$

$$\begin{cases} 64a + 16b + 4c - 16 = 0 \\ 27a + 9b + 3c - 16 = 5 \\ a + b + c - 16 = -9 \end{cases}$$

$P(0) = a(0)^3 + b(0)^2 + c(0) + d = -16 \Rightarrow \boxed{d = -16}$

$$\Rightarrow \begin{cases} 64a + 16b + 4c = 16 \\ 27a + 9b + 3c = 5 + 16 \\ a + b + c = -9 + 16 \end{cases} \xRightarrow{\text{SIMPLIFICANDO}} \begin{cases} 16a + 4b + c = 4 & (I) \\ 9a + 3b + c = 7 & (II) \\ a + b + c = 7 & (III) \end{cases}$$

$I - II \rightarrow 7a + b = -3 \xrightarrow{-2}$
$II - III \rightarrow 8a + 2b = 0$

$$\begin{cases} -14a - 2b = 6 \\ 8a + 2b = 0 \end{cases}$$
$-6a = 6$ $\quad a = -1$

$\Rightarrow$ DESPEJANDO $b = 4$ $\quad c = 4$

$$\boxed{P(x) = -x^3 + 4x^2 + 4x - 16}$$

VERIFICACIÓN:

	-1	4	4	-16
4		-4	0	16
	-1	0	4	$\lfloor 0$ ✓

	-1	4	4	-16
3		-3	3	21
	-1	1	7	$\lfloor 5$ ✓

	-1	4	4	-16
1		-1	3	7
	-1	3	7	$\lfloor -9$ ✓

UTU Escuela Técnica Buceo Examen de Matemática 9° 11/07/2008

RESOLUCIÓN

1) ii) $P(x) = -x^3 + 4x^2 + 4x - 16$

$$\begin{array}{r|rrrr} & -1 & 4 & 4 & -16 \\ 4 & & -4 & 0 & 16 \\ \hline & -1 & 0 & 4 & \boxed{0} \end{array}$$

$-x^2 + 4 = 0$ RAÍCES 2 y -2.

$P(x) = -1(x-4)(x-2)(x+2)$

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2) i) CAMINERO $\longrightarrow$ 8 LETRAS : C M N R 4 CONSONANTES; A I E O 4 VOCALES

$A^8_6 = 8 \times 7 \times 6 \times 5 \times 4 \times 3 = 20160$

ii) $\left.\begin{array}{l}\text{4 CONSONANTES} \\ \text{2 VOCALES}\end{array}\right\} \longrightarrow \left\{\begin{array}{ll}\text{ELEGIR 2 VOCALES DE 4 DISPONIBLES} & C^4_2 \\ \text{" 4 CONSON. DE 4 "} & C^4_4 \\ \text{Y LUEGO PERMUTAMOS LAS 6 LETRAS} & \end{array}\right.$

RESPUESTA $\longrightarrow$ $C^4_2 \cdot \underbrace{C^4_4}_{1} \cdot P_6 = \dfrac{4 \times 3}{2} \cdot 6! = 4320$

iii)

* M I N _ _ _ $\longrightarrow$ NOS FALTAN 2 CONSONANTES y 1 VOCAL

* _ M I N _ _

* _ _ M I N _

* _ _ _ M I N

$\left\{\begin{array}{ll}\text{ELEGIR 1 VOCAL DE 3 DISPONIBLES} & C^3_1 \\ \text{PERMUTAR LAS 3 LETRAS} & \end{array}\right.$

$C^3_1 \cdot P_3 \cdot 4 = 3 \cdot 3! \cdot 4 = 72$

* 4 OPCIONES

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5)

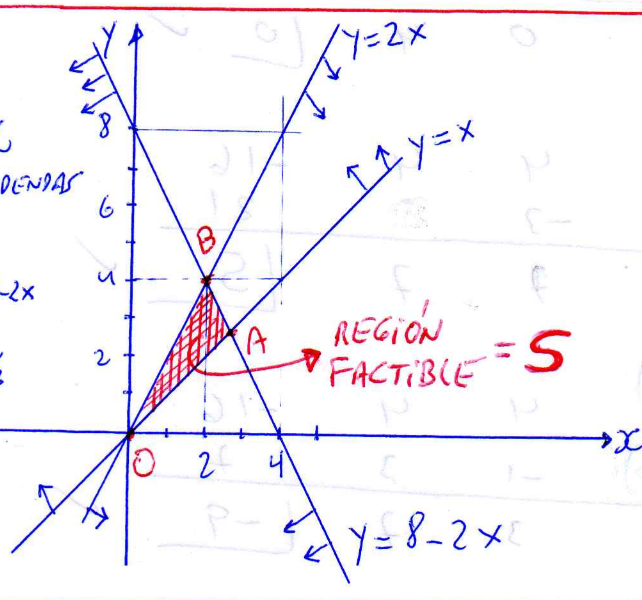
DETERMINACIÓN DE LAS COORDENADAS DE A:

$\left.\begin{array}{l} y = x \\ y = 8 - 2x \end{array}\right\} \rightarrow x = 8 - 2x$

$3x = 8$

$x = 8/3$

$y = x \Rightarrow y = 8/3$



PUNTO	x	y	$z = x + y$
O	0	0	$z = 0 + 0 = 0$
A	$\frac{8}{3}$	$\frac{8}{3}$	$z = \frac{8}{3} + \frac{8}{3} = \frac{16}{3} \approx 5{,}3\overline{3}$
B	2	4	$z = 2 + 4 = 6$ ←

MÁXIMO: EN B

MÁXIMO = $\boxed{z = 6}$ ✓

UTU Escuela Técnica Buceo Examen de Matemática 9° 11/07/2008

RESOLUCIÓN

3) i) $\frac{(m+1)!}{(m+3)!} = \frac{\cancel{1}.\cancel{2}.\cancel{3}.\cancel{\ldots}.\cancel{(m+1)}}{\cancel{1}.\cancel{2}.\cancel{3}.\cancel{\ldots}.\cancel{(m+1)}(m+2)(m+3)} = \frac{1}{(m+2)(m+3)} = \frac{1}{42} \Leftrightarrow$

$\Leftrightarrow (m+2)(m+3) = 42 \Leftrightarrow m = 4$ ✓

$m = -9$ ✗

ii) $A_3^m = 20m$

$\cancel{m}(m-1)(m-2) = 20\cancel{m}$ $m=0$ ✗

$(m-1)(m-2) = 20$ $m = 6$ ✓

$m = -3$ ✗

iii) $C_{m+3}^{m^2-2m} = C_{3m+4}^{m^2-2m}$

IDÉNTICAS

$m+3 = 3m+4$

$3-4 = 3m-m$

$-1 = 2m$

$-\frac{1}{2} = m$ ✗

NO VERIFICA

COMPLEMENTARIAS

$m+3+3m+4 = m^2-2m$

$0 = m^2-6m-7$ → $m=7$, $m=-1$

$m=7$ $C_{10}^{35} = C_{25}^{35}$ ✓

$m=-1$ $C_2^3 = C_1^3$ ✓

RESOLUCIÓN

4) * $P(x) = (x^2+1)(ax^2+bx+c)$

$$\begin{array}{l|l} P(x) & x^2+4x+3 \\ \hline -274x-262 & Q(x) \end{array}$$

$P(x) = (x^2+4x+3)\cdot Q(x) - 274x - 262$

$P(1) = -40$

$x^2+4x+3=0 \quad x=-1,\ x=-3$

$P(-1) = \underbrace{((-1)^2+4(-1)+3)}_{=0}\cdot Q(-1) - 274(-1) - 262 = 12$

$P(-3) = \underbrace{((-3)^2+4(-3)+3)}_{=0}\cdot Q(-3) - 274(-3) - 262 = 560$

ENTONCES: SUSTITUIMOS EN *

$P(1) = (1^2+1)(a+b+c) = -40 \longrightarrow a+b+c = -\frac{40}{2}$

$P(-1) = ((-1)^2+1)(a-b+c) = 12 \longrightarrow a-b+c = \frac{12}{2}$

$P(-3) = ((-3)^2+1)(9a-3b+c) = 560 \longrightarrow 9a-3b+c = \frac{560}{10}$

$$\left\{\begin{array}{l} a+b+c=-20 \\ a-b+c = 6 \\ 9a-3b+c = 56 \end{array}\right\} \Longrightarrow \begin{array}{l} a=3 \\ b=-13 \\ c=-10 \end{array}$$

RESPUESTA: $\boxed{P(x) = (x^2+1)(3x^2-13x-10)}$

VAMOS A DESARROLLAR: $P(x) = 3x^4-13x^3-10x^2+3x^2-13x-10$

$\boxed{P(x) = 3x^4-13x^3-7x^2-13x-10}$ ✓

VERIF:

	3	-13	-7	-13	-10
-3		-9	66	-177	570
	3	-22	59	-190	560

	3	-13	-7	-13	-10
-1		-3	16	-9	22
	3	-16	9	-22	12