Ejercicio 7

i) Probar que $\sum_{i=0}^{n} C_i^n$ es múltiplo de 2 $\forall n, \quad n \in \mathbb{N}$

ii) Calcular $\sum_{k=0}^{100} \left(\frac{1}{k+2} - \frac{1}{k+1}\right)$ en forma exacta.

i) i=0 C(0,0) = 1 = 2^0
i=1 C(1,0)+C(1,1) = 1+1 = 2^1
i=2 C(2,0)+C(2,1)+C(2,2) = 1+2+1 = 4 = 2^2

H) n=h $\sum_{i=0}^{h} C_i$ = C(h,0)+C(h,1)+C(h,2)+ ……+C(h,h) = 2^h

T) n=h+1 $\sum_{i=0}^{h+1} C_i$ =C(h+1,0)+C(h+1,1)+C(h+1,2)+…..C(h+1,h+1) = 2^{h+1}

C(h+1,0) = C(h,0)
Por Stieffel:
C(h+1,1) = C(h,0)+C(h,1)
C(h+1,2) = C(h,1)+C(h,2)
C(h+1,3) = C(h,2)+C(h,3)
. . .
C(h+1,h) = C(h,h-1)+C(h,h)
C(h+1,h+1) = C(h,h) Entonces $\sum_{i=0}^{h+1} C_i$ = 2 x 2^h = 2^{h+1}

ii) $\sum_{k=0}^{100} \left(\frac{1}{k+2} - \frac{1}{k+1}\right)$

Para k=0 tenemos: (1/2-1/1) = 1/2-1= -1/2
Para k=1 -1/2+(1/3-1/2) = 1/3-1/2-1/2 = 1/3-1= -2/3
Para k=2 -2/3+(1/4-1/3) = 1/4-2/3-1/3 = 1/4-3/3 = 1/4-1 = -3/4
Para k=3 -3/4+(1/5-1/4) = 1/5-3/4-1/4 = 1/5-4/4 = 1/5-1 = -4/5

Vemos entonces que los términos de la sumatoria van tomando la forma:

-1/2,-2/3,-3/4,-4/5, o sea -(k+1)/(k+2).
Entonces para k=100 el valor habrá de ser -101/102

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